Name: Key

- This packet is due at **12:00 pm PDT (noon) on ~~Friday, May 21~~**.
- Neatly fill out this packet either directly or on your own paper. Then upload your answers to Canvas. Please clearly number each problem and keep them in order.
- For each problem in this packet, **organize and show all of your work and justification** for your answer.
- Make sure that it is clear what your final answer is. This may mean boxing/circling it or it may mean doing neither.
- If you are required to draw a graph of a function, each axis must be labeled and given a scale.
- I will not be grading every individual problem, but answers will be available on Canvas after the due date.
- I am happy to help you however I can. Here are some options:
  - Come to my office hours (see syllabus).
  - Email me directly at gknapp4@uoregon.edu

1. Find the Taylor polynomial of degree 2 for $f(x) = \sqrt{x}$ centered at $a = 4$.

$f(4) = 2 \longrightarrow c_0 = 2$

$f'(x) = \frac{1}{2}x^{-1/2} \longrightarrow f'(4) = \frac{1}{4} \longrightarrow c_1 = \frac{1}{4}$

$f''(x) = \frac{-1}{4}x^{-3/2} \longrightarrow f''(4) = \frac{-1}{32} \longrightarrow c_2 = \frac{-1}{64}$

$\Rightarrow p_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$

2. Compute the Taylor series of $g(x) = x^3$ centered at $a = 1$

$g(1) = 1 \longrightarrow c_0 = 1$

$g'(x) = 3x^2 \longrightarrow g'(1) = 3 \longrightarrow c_1 = 3$

$g''(x) = 6x \longrightarrow g''(1) = 6 \longrightarrow c_2 = 3$

$g'''(x) = 6 \longrightarrow g'''(1) = 6 \longrightarrow c_3 = 1$

$g^{(n)}(x) = 0$ for $n \geq 4 \longrightarrow c_n = 0$ for $n \geq 4$

$\Rightarrow$ Taylor series for $g(x) = x^3$ centered at 1 is $1 + 3(x-1) + 3(x-1)^2 + (x-1)^3$

3. Find a formula for the nth Taylor coefficient for $f(x) = \sin(x)$ at $a = 0$.

$f(x) = \sin(x)$, $f'(x) = \cos(x)$, $f''(x) = -\sin(x)$, $f'''(x) = -\cos(x)$

$f^{(4)}(x) = \sin(x)$, ...

$$\rightarrow f^{(n)}(x) = \begin{cases} \sin(x) & n = 4k \\ \cos(x) & n = 4k+1 \\ -\sin(x) & n = 4k+2 \\ -\cos(x) & n = 4k+3 \end{cases}$$

$$\rightarrow f^{(n)}(0) = \begin{cases} 0 & n = 4k \\ 1 & n = 4k+1 \\ 0 & n = 4k+2 \\ -1 & n = 4k+3 \end{cases}$$

$$\rightarrow C_n = \begin{cases} 0 & n = 4k \\ 1/n! & n = 4k+1 \\ 0 & n = 4k+2 \\ -1/n! & n = 4k+3 \end{cases}$$

4. Use the Maclaurin Series for $f(x) = e^x$ to estimate e^{-4} with error < 0.0001.

Note: $f^{(n)}(x) = e^x$ for all n

$\Rightarrow |f^{(n)}(x)| \leq 1$ on $[-4, 0]$

$\Rightarrow |R_n(-4)| \leq \frac{1}{(n+1)!} \cdot |-4-0|^{n+1} = \frac{4^{n+1}}{(n+1)!}$

Want: $\frac{4^{n+1}}{(n+1)!} < 0.0001$

Trial and Error: $n = 16$ works

$\Rightarrow e^{-4} \approx \sum_{n=0}^{16} \frac{(-4)^n}{n!} \approx 0.0183$